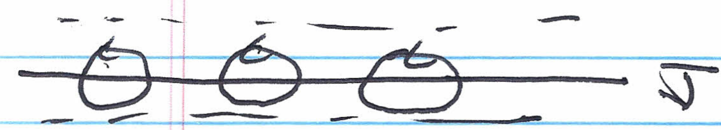
Physics 87

Class 2 — Toroidal Confinement I

Burn → $n T \tau_E > \text{crit}$

— Recall late 40's : Z-pinch



(continues today; Sandia ⇒ pulsed)

⇒ $F_r = \frac{J_z B_\theta}{c} \sim J_z^2$

→ heating : Ohmic ; $\partial_t T = \underline{E} \cdot \underline{J}$

⇒ T $\qquad = \sigma E^2$

→ confinement : B_θ

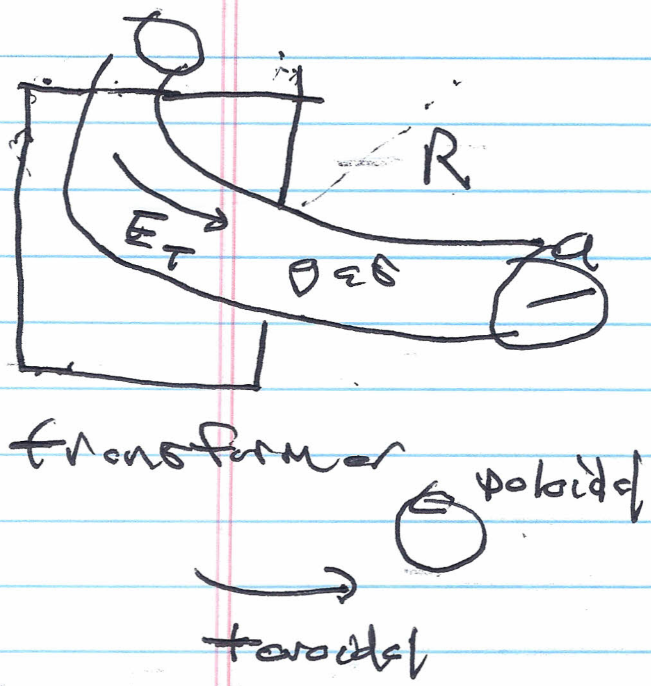
⇒ τ_E

→ compression : F_r

⇒ n

but : stability ! ?

— simple improvement : { toroidal pinch
RFP (Reversed Field Pinch)



simple toroidal system:

- a, R
  $a/R \sim 1/4, 1/5$
- transformer, with plasma as secondary

$\Rightarrow E_T, J_T, B_\theta$

- fill pressure of gas, ionized.

start - mid, late 50's

notable: Zeta, U.K.
early 60's

continues: RFX, Padova
MST, Madison

- Toroidal Pinch is story of
  → stability (macro)
  → self-organization.

Stability:

- Can plasma lower its energy by displacement?
- consequences: very poor confinement, disruption, short lifetime

Simplest Model: MHD (Magnetohydrodynamics)

Fluid (neutral): Navier-Stokes

$$\rho\left(\frac{\partial \underline{v}}{\partial t} + \underline{v}\cdot\underline{\nabla}\,\underline{v}\right) = -\underline{\nabla} P + \underline{f}_{body}$$

$$P = P(n, T)$$

(don't scare or impress)

Magnetofluid: - net neutral
- +, -

$$\partial_t \rho + \underline{\nabla}\cdot(\rho \underline{v}) = 0$$

(continuity)

Lorentz force.

$$\rho\left(\partial_t \underline{v} + \underline{v}\cdot\underline{\nabla}\,\underline{v}\right) = -\underline{\nabla} p + \frac{\underline{J}\times\underline{B}}{c} + \underline{f}_{body}$$

magn. pressure   magn. tension

$$= -\underline{\nabla}\left(p + \frac{B^2}{8\pi}\right) + \frac{\underline{B}\cdot\underline{\nabla}\,\underline{B}}{4\pi} + \underline{f}_{body}$$

Momentum Balance → replaces Navier–Stokes.
N.B.: electrically neutral fluid, no $q\underline{E}$ force.

$$\underline{E} + \frac{\underline{v}\times\underline{B}}{c} = \cancel{\eta}\,\underline{J}$$

Ohm's Law

ideal

moving plasma

N.B.: $\left\{\begin{array}{l}\underline{v} \to \text{ions}\\ \underline{J} \to \text{electrons}\end{array}\right.$

and modified Maxwell Eqns:

$$\underline{\nabla}\cdot\underline{B} = 0$$ (no monopoles)

$$\underline{\nabla}\times\underline{B} = \frac{4\pi}{c}\underline{J}$$

(Ampere's law; low freq. phenomena so neglect displacement current)

$$\underline{\nabla}\times\underline{E} = -\frac{1}{c}\frac{\partial \underline{B}}{\partial t}$$

Faraday's Law.

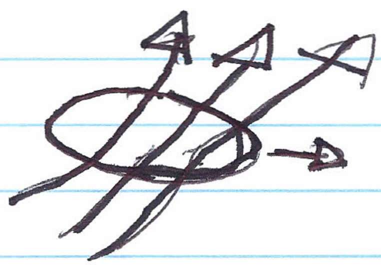
→ Important Property

for $\underline{E} + \frac{\underline{v} \times \underline{B}}{c} = 0$

[ideal MHD fluid
$\eta^{-1} = \sigma \to \infty$

field frozen into fluid:

$$\frac{d}{dt}\int_C \underline{B} \cdot d\underline{a} = 0$$



flux thru moving loop constant!

⇒ MHD is two interpenetrating 'fluids' of $\vec{v}$ and $\underline{B}$ frozen into each other

⇒ resistivity breaks freezing-in law.

Freezing in ⟺ Alfven wave.

Neutrality — no non-trivial Gauss Law and eqn of state.

→ [redacted] 7 equations!

→ For stability, can derive change in energy, for displacement $\underline{\xi}$, where $\partial_t \underline{\xi} = \underline{V}$.

$\underline{\xi}$ is displacement from stationary equilibrium.

$$\delta W = \frac{1}{2}\int d^3x \left[\frac{(\underline{\nabla}\times\underline{\xi}\times\underline{B}_0)^2}{4\pi} + \underline{J}_0\cdot\underline{\xi}\times\underline{Q} + \gamma\, p_0 (\underline{\nabla}\cdot\underline{\xi})^2 + (\underline{\xi}\cdot\underline{\nabla} p_0)(\underline{\nabla}\cdot\underline{\xi}) - (\underline{\xi}\cdot\underline{\nabla}\phi)\,\underline{\nabla}\cdot(\rho\,\underline{\xi}) \right]$$

change in potential energy

$$\underline{Q} = \underline{\nabla}\times\underline{\xi}\times\underline{B}_0$$

$$\underline{F}_g = \underline{\nabla}\phi$$

— 2nd order in $\underline{\xi}$

— a la variational principle

(+) terms : $\delta W > 0$ → stable

$\underline{\sigma}$, $\delta W < 0$ → instability
→ due to

$\underline{J_0}$ → current

$\underline{\nabla} P_0$ → pressure gradient

$\underline{g} \cdot \underline{\nabla} \rho_0$ → density gradient

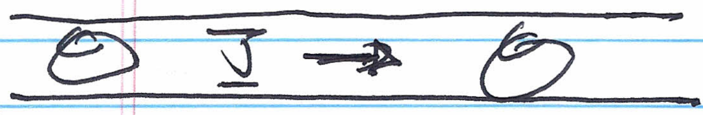
For pinch : Current is the key.

→ Simple picture of two current-driven instabilities related to pinch.

- sausage instability
- kink instability

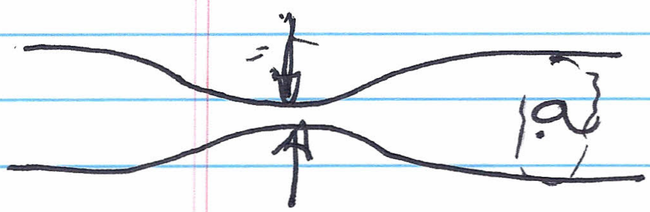
a) Sausage

Pinch ≡ periodic cylinder



Toroidal current → poloidal field

squeeze:



now: $F_r = \frac{J_z}{c} B_\theta$

$= J_z \frac{2I}{r}$

$I = \int J \cdot d\varepsilon$

due freezing in

so if $r \to r - \Delta r$

$F_r' > F_r$

$\Rightarrow$

- initially current pinches itself

- perturbing current compresses plasma, strengthens pinch, enhances F_n

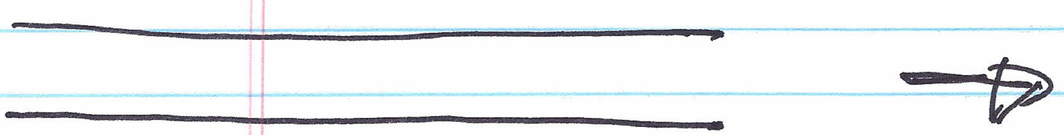
$\Rightarrow$

- self-re-enforcing feedback loop
  $\Rightarrow$ instability!

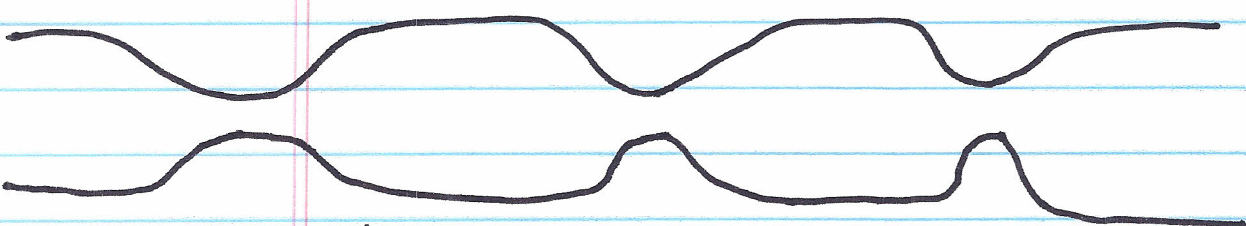
pinch $\rightarrow$ squeeze $\longrightarrow$ compression

compression $\rightarrow$ enhanced squeeze $\rightarrow$ squeeze

upshot: 'Sausage' instability!



$\tau \sim 10-100$ μsec.



a la' chain of links...

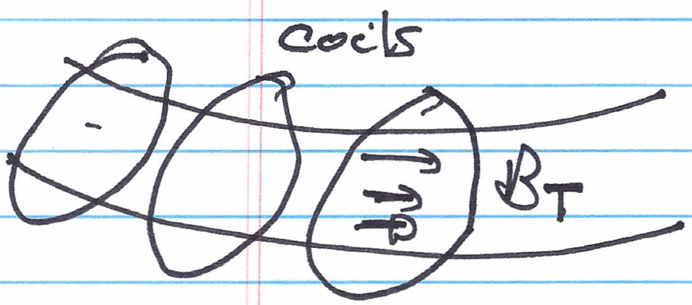
driven by <u>magnetic tension</u> $\Leftrightarrow$ current

→ akin to line of water breaking up into chain of droplets

due surface tension.

→ Sausage was first instability encountered by Pinch research.

→ Remedy: toroidal field



(vacuum: not carried by plasma current)

Question: – Estimate B_T to stabilize sausage, for I_p

– $F_r \uparrow$ vs opposite $F_r \uparrow$ due compression of toroidal field.

Also: Read Roberts, P. D. on MHD; posted.

→ But.... enter the kink!

- With B_z, magnetic field line winds, helically

- magnetic tension ⟺ kink

Line:

$$\frac{dz}{B_z} = \frac{r\,d\theta}{B_\theta} = \frac{dr}{B_r}$$

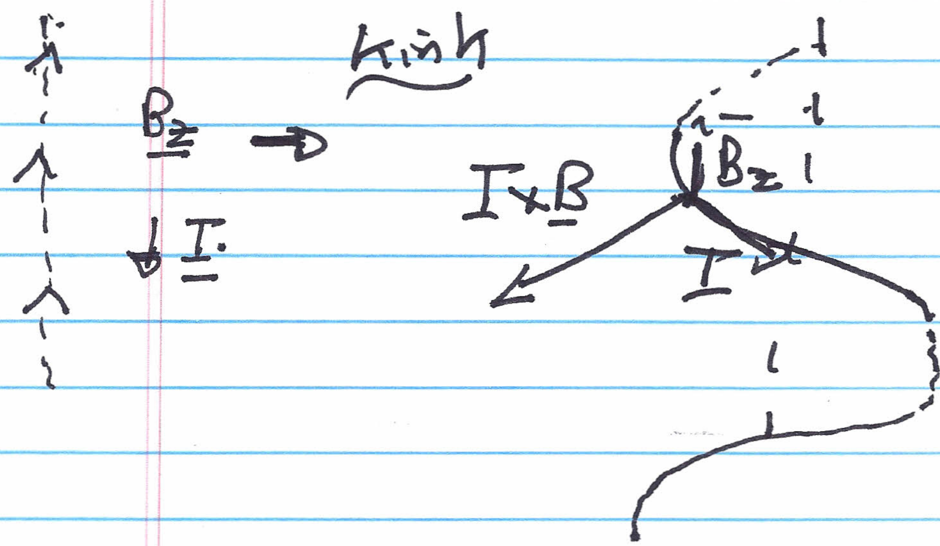
$$r\frac{d\theta}{dz} = \frac{B_\theta}{B_z}$$

$$\frac{r}{R}\frac{d\theta}{d\phi} = \frac{B_\theta}{B_z}$$

$$\frac{d\theta}{d\phi} = \frac{R}{r}\frac{B_\theta}{B_z} \equiv \frac{1}{q(r)}$$

pitch of line

Kink occurs from $\underline{J} \times \underline{B}$ force on wire,



obviously $\underline{B}_z$ essential to kink dynamics

$\therefore$ $\underline{I} \times \underline{B}$ force reinforces initial kink perturbation

$\Rightarrow$

- simple pinch $\Rightarrow$ sausage
- introducing B_z to kill sausage $\Rightarrow$ kink!

Pinches struggled with MHD stability until Zeta, early 60's

Stability problems:

→ short life

→ bad σ_E, low T

→ high turbulence.

TBC next week.